WESLEY COLLEGE

By daring & by doing

YEAR 12 2018 MATHEMATICS METHODS

CRV's & Normal Probability

Test 5

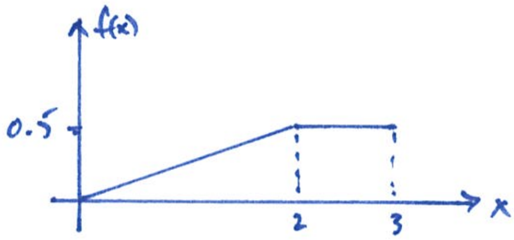
Name: Solutions. **Marks:** /50

Calculator Free (19 marks)

1. [2, 2, 3 = 7 marks]

For each of the following, state whether it represents a continuous probability distribution. Briefly justify your answers.

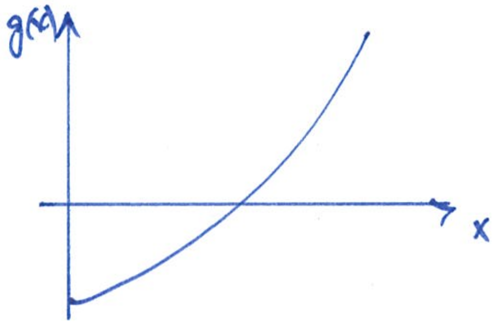
a) $$f(x)=\begin{cases}\frac{1}{4}x & , 0\le x\le 2\\ \frac{1}{2} & , 2<x\le 3\\ 0 & , elsewhere\end{cases}$$



Area $= 0.5\times 2\times 0.5 + 0.5\times 1 = 1$

$\therefore$ Yes.

b) $$g(x)=\begin{cases}\frac{7}{9}x^2-2, & 0\le x\le 3\\ 0 & , elsewhere\end{cases}$$

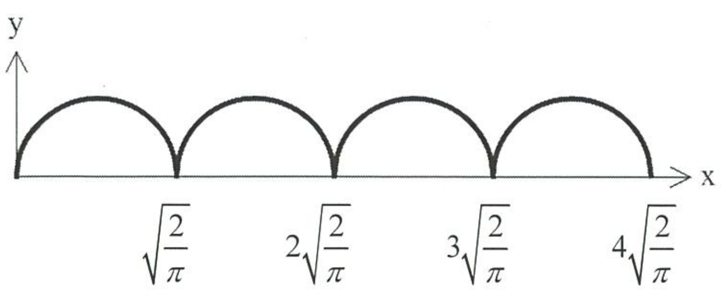


Under x axis

$\therefore$ No!

Yes/No only 0 marks

c)



$r=\frac{1}{2}\sqrt{\frac{2}{\pi}}$

$\therefore$ Yes.

Area $= 2\times\pi\times\left(\frac{1}{2}\sqrt{\frac{2}{\pi}}\right)^2$

$= 2\times\pi\times\frac{1}{4}\times\frac{2}{\pi} = 1$

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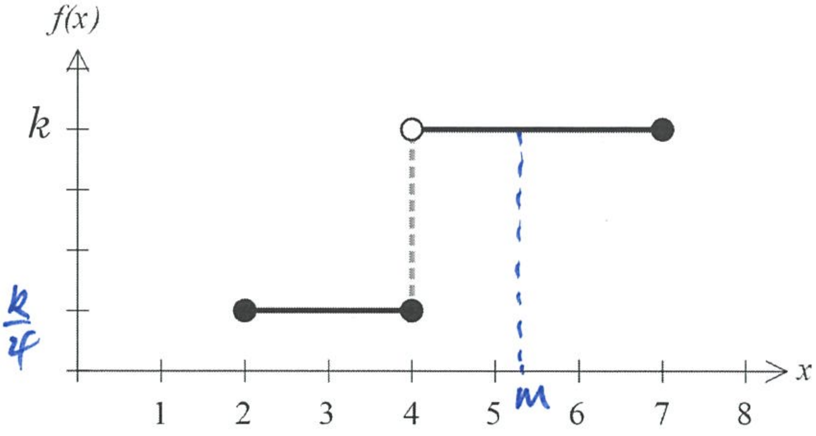
2. [2, 3 = 5 marks]

The graph below is a probability density function for a random variable X.

$$f(x) = \begin{cases} 0 & , x < 2 \\ \dfrac{k}{4} & , 2 \le x \le 4 \\ k & , 4 < x \le 7 \\ 0 & , x > 7 \end{cases}$$



a) Determine the value of k.

Area = 1 $\Rightarrow$ $2 \times \frac{k}{4} + 3 \times k = 1$ (1)

$\frac{k}{2} + 3k = 1$

$7k = 2$

$k = \frac{2}{7}$ (1)

b) Determine the median.

$\therefore$ area from $m \to 7 = 0.5$ (1)

$(7-m)\frac{2}{7} = 0.5$ (1)

$7 - m = \frac{7}{4}$

$m = 7 - \frac{7}{4}$ (1)

$= 5\frac{1}{4}$ or 5.25

$\overline{5}$

3. **[1, 2, 2, 2 = 7 marks]**

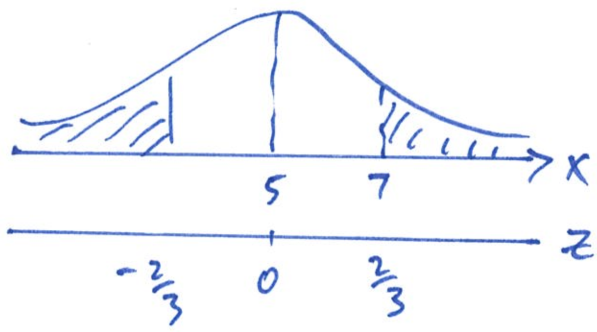
a) Let X be a normally distributed random variable with mean 5 and variance 9 and let Z be a random variable with the standard normal distribution.
Determine;

(i) $P(X > 5)$ $\quad 0.5 \quad (1)$

(ii) b such that $P(X > 7) = P(Z < b)$

$Z = \frac{7-5}{3} = \frac{2}{3} \quad (1)$

$\therefore b = -\frac{2}{3} \quad (1)$



b) A golfer believes that the distance, in metres, that he hits a ball with a 7-iron, follows a uniform distribution over the interval [100, 150].

(i) Determine the median and interquartile range of the distance he hits a ball that would be predicted by this model.

Median = 125 m (1)

IQR = 25 m (1)

(ii) Explain why the continuous uniform distribution may not be a suitable model.

Would expect most shots to around the median of 125 m and not equally distributed between 100m → 150m (2)

End of Part A

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NAME: ______________________________

__Calculator Section__ **(31 marks)**

4. [2, 2 = 4 marks]

Flaboway is a company that operates fitness centres (gyms) across Australia. Members are assessed each month by undergoing a set of exercises called JIM.

The company has found that the time taken by members to complete JIM is a continuous random variable X, with probability distribution function g:

$$g(x) = \begin{cases} \dfrac{(x-3)^3+64}{256} & ,1 \le x \le 3 \\ \dfrac{x+29}{128} & ,3 < x \le 5 \\ 0 & ,elsewhere \end{cases}$$

a) Evaluate $E(X)$, correct to four decimal places. (1)

$$E(x) = \int_1^3 \left(x \times \frac{(x-3)^3+64}{256}\right) dx + \int_3^5 \left(x \times \frac{x+29}{128}\right) dx$$

$$= 3.0458$$

[Can you DEFINE function] 4 dp (1)

b) In a random sample of 200 Flaboway members, how many members would be expected to take more than four minutes to complete JIM?

$$\int_4^5 \frac{x+29}{128}\, dx \times 200 = 52.34 \quad (1)$$

$\therefore$ 52 members. (1)

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5. [2, 1, 2, 3 = 8 marks]

A machine is set to fill packets of potato chips with 200g of chips. However, due to inaccuracy of this type of machine, the actual weights in packets are normally distributed with a mean of 201g and a standard deviation of 4.5g. A quality control measure used by the factory is to weigh each packet after filling and recycle any packet less than 195g.

$X \sim N(201, 4.5^2)$

a) Determine the percentage of packets will be recycled.

$P(X < 195) = 0.0912$ (1)

$\sim 9.12\%$ (1)

b) If the factory produces 12 000 packets per day, how many will be recycled in one day?

$0.0912 \times 12000 = 1094.4$

~ 1094 (1) check back to a) rounding.

c) If a packet is selected from those destined for recycling, what is the probability that its weight is less than 190g?

$P(X < 190 \mid X < 195)$

$= \dfrac{P(X < 190)}{P(X < 195)}$ (1)

$= \dfrac{0.0073}{0.0912} = 0.08$ (1) (0.0795)

d) If the company wishes to reduce the percentage recycled to 5% and can do this by increasing the mean without affecting the standard deviation, what mean weight should they set? Show working to justify your answer.

Want $P(X < 195) = 0.05$

So $\dfrac{195 - \bar{x}}{4.5} = -1.644$

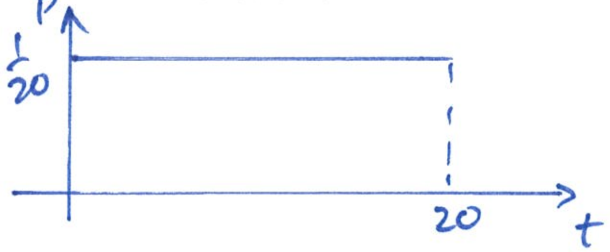
$\therefore \bar{x} = 202.4$ g

$\frac{}{8}$

6. [2, 2, 1, 2, 2 = 9 marks]

A leaking tap releases a drop of water every 20 seconds. A bird comes every day to drink at this tap, arriving at a random time and taking the first drop of water.

a) Let T represent the time the bird must wait to drink. Draw the graph of a suitable probability distribution for the random variable T.



(2)

b) Determine the probability that, next Monday, the bird will have to wait at least 15 seconds to drink.

$P(T > 15) = 5 \times \frac{1}{20} = \frac{1}{4}$ (2)

c) Determine the probability that, next Wednesday; the bird will have to wait exactly 15 seconds to drink.

Continuous $\therefore$ not possible (1)

d) Determine the probability that, next week, the bird will have to wait at least 15 seconds to drink on exactly four days

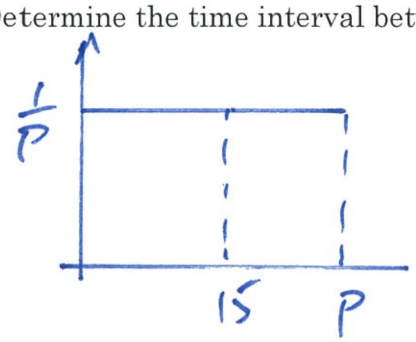
$Y \sim B(7, \frac{1}{4})$ (1)

$P(Y=4) = 0.05768$ (1)

~ 0.06

At another leaking tap, the probability that the bird has to wait at least 15 seconds is 0.8

e) Determine the time interval between successive drops.



So $15 \times \frac{1}{p} = \frac{1}{5}$ (1)

$\frac{1}{p} = \frac{1}{75}$

$\therefore p = 75s.$ (1)

$\therefore$ Time between drops is 75s.

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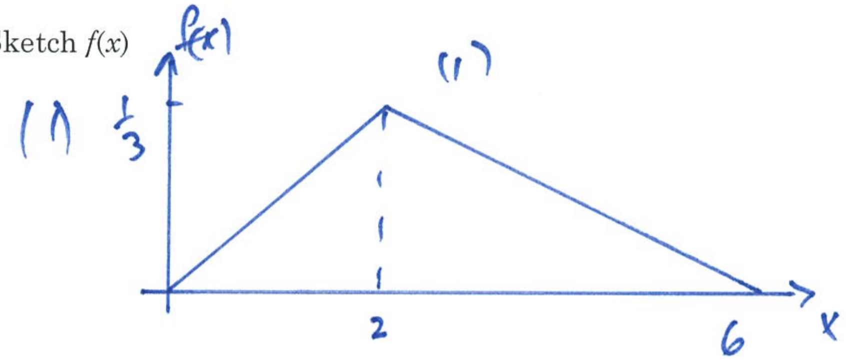
7. [3, 7 = 10 marks]

The continuous random variable X has the following PDF:

$$f(x) = \begin{cases} \frac{1}{6}x & , 0 \le x \le 2 \\ \frac{1}{12}(6-x) & , 2 < x \le 6 \\ 0 & , elsewhere \end{cases}$$

a) Sketch $f(x)$



(1)

b) Define fully the cumulative distribution function $F(x)$ of X.

$$F(x) = \begin{cases} 0 & , x < 0 \\ \frac{1}{12}x^2 & , 0 \le x \le 2 \\ \frac{1}{3} + \frac{(2-t)(t-10)}{24} & , 2 < x \le 6 \\ 1 & , x > 6 \end{cases}$$

(1)

(2)

(4)

part marks for working

$$\frac{1}{3} + \int_2^m \frac{1}{12}(6-x)\,dx$$

$$= \frac{1}{3} + \frac{(2-m)(m-10)}{12}$$

$$=$$

End of Part B

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